Lösungen der Hausaufgabe Nr. 10 Lineare Algebra Studiengang Network Computing WS 2004/2005

Martin Grandrath (Matr. Nr.: 46375)

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1 Permutation

$$p = (1\ 4\ 2\ 9\ 3\ 8) \circ (5\ 7)$$

Zyklische Untergruppe $\langle p \rangle$

$$\begin{aligned} e &= p^m = (p_1 \circ p_2)^m = p_1^m \circ p_2^m \\ m &= 6k_1 \ \wedge \ m = 2k_2 \\ m &= \text{kgV}(6,2) = 6 \end{aligned}$$

$\curvearrowright |\langle p \rangle| = 6$

p ist darstellbar als Produkt von $5 + 1 = 6$ Transpositionen.
$\curvearrowright p$ ist gerade.

2 Normalteiler

Erstellen Cayley-Tafel:

$$
\begin{aligned}
p \circ p &= (1\ 2\ 3) \circ (1\ 2\ 3) = (1\ 3\ 2) = q \\
p \circ q &= (1\ 2\ 3) \circ (1\ 3\ 2) = e \\
p \circ r &= (1\ 2\ 3) \circ (2\ 3) = (1\ 3) = t \\
p \circ s &= (1\ 2\ 3) \circ (1\ 2) = (2\ 3) = r \\
p \circ t &= (1\ 2\ 3) \circ (1\ 3) = (1\ 2) = s \\
\\
q \circ p &= (1\ 3\ 2) \circ (1\ 2\ 3) = e \\
q \circ q &= (1\ 3\ 2) \circ (1\ 3\ 2) = (1\ 2\ 3) = p \\
q \circ r &= (1\ 3\ 2) \circ (2\ 3) = (1\ 2) = s \\
q \circ s &= (1\ 3\ 2) \circ (1\ 2) = (1\ 3) = t \\
q \circ t &= (1\ 3\ 2) \circ (1\ 3) = (2\ 3) = r \\
\\
r \circ p &= (2\ 3) \circ (1\ 2\ 3) = (1\ 2) = s \\
r \circ q &= (2\ 3) \circ (1\ 3\ 2) = (1\ 3) = t \\
r \circ r &= (2\ 3) \circ (2\ 3) = e \\
r \circ s &= (2\ 3) \circ (1\ 2) = (1\ 2\ 3) = p \\
r \circ t &= (2\ 3) \circ (1\ 3) = (1\ 3\ 2) = q \\
\\
s \circ p &= (1\ 2) \circ (1\ 2\ 3) = (1\ 3) = t \\
s \circ q &= (1\ 2) \circ (1\ 3\ 2) = (2\ 3) = r \\
s \circ r &= (1\ 2) \circ (2\ 3) = (1\ 3\ 2) = q \\
s \circ s &= (1\ 2) \circ (1\ 2) = e \\
s \circ t &= (1\ 2) \circ (1\ 3) = (1\ 2\ 3) = p \\
\\
t \circ p &= (1\ 3) \circ (1\ 2\ 3) = (2\ 3) = r \\
t \circ q &= (1\ 3) \circ (1\ 3\ 2) = (1\ 2) = s \\
t \circ r &= (1\ 3) \circ (2\ 3) = (1\ 2\ 3) = p \\
t \circ s &= (1\ 3) \circ (1\ 2) = (1\ 3\ 2) = q \\
t \circ t &= (1\ 3) \circ (1\ 3) = e
\end{aligned}
$$

Cayley-Tafel:

$\circ$	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	t	r	s
q	q	e	p	s	t	r
r	r	s	t	e	p	q
s	s	t	r	q	e	p
t	t	r	s	p	q	e

zu zeigen:

$$\forall a \in S_3 : \; a\langle p\rangle = \langle p\rangle a$$

- $a = e$

$$\begin{aligned} e\langle p\rangle &= \{ee, ep, eq\} \\ &= \{e, p, q\} \\ \langle p\rangle e &= \{ee, pe, qe\} \\ &= \{e, p, q\} \end{aligned}$$

- $a = p$

$$\begin{aligned} p\langle p\rangle &= \{pe, pp, pq\} \\ &= \{p, q, e\} \\ \langle p\rangle p &= \{ep, pp, qp\} \\ &= \{p, q, e\} \end{aligned}$$

- $a = q$

$$\begin{aligned} q\langle p\rangle &= \{qe, qp, qq\} \\ &= \{q, e, p\} \\ \langle p\rangle q &= \{eq, pq, qq\} \\ &= \{q, e, p\} \end{aligned}$$

- $a = r$

$$\begin{aligned} r\langle p\rangle &= \{re, rp, rq\} \\ &= \{r, s, t\} \\ \langle p\rangle r &= \{er, pr, qr\} \\ &= \{r, t, s\} \end{aligned}$$

- $a = s$

$$\begin{aligned} s\langle p\rangle &= \{se, sp, sq\} \\ &= \{s, t, r\} \\ \langle p\rangle s &= \{es, ps, qs\} \\ &= \{s, r, t\} \end{aligned}$$

- $a = t$

$$\begin{aligned} t\langle p\rangle &= \{te, tp, tq\} \\ &= \{t, r, s\} \\ \langle p\rangle t &= \{et, pt, qt\} \\ &= \{t, s, r\} \end{aligned}$$

$\curvearrowright$ $\langle p\rangle$ ist Normalteiler von S_3.

Faktorgruppe

$$\begin{aligned} S_3/_{\langle p\rangle} &= \{a\langle p\rangle : \ a \in S_3\} \\ &= \Big\{ \underbrace{\{e, p, q\}}_{N}, \underbrace{\{r, s, t\}}_{M} \Big\} \end{aligned}$$

Cayley-Tafel der Faktorgruppe

$\circ$	N	M
N	N	M
M	M	N