Lösungen der Hausaufgabe Nr. 7 Lineare Algebra Studiengang Network Computing WS 2004/2005

Martin Grandrath (Matr. Nr.: 46375)

2. Dezember 2004

1 Supremum und Infimum

$$A = \left\{ \frac{n}{n-1} : \ n \in \mathbb{N} \setminus \{0,1\} \right\}$$
$$\curvearrowright \forall a \in A : \ a \in (1,2]$$

$$\sup_{\mathbb{R}} A = 2$$
$$\inf_{\mathbb{R}} A = 1$$

2 Matrizen

2.1

$$\begin{array}{ccc|cc} & & & 1 & 2 \\ & & & 0 & 6 \\ & & & 1 & 2 \\ \hline 1 & 1 & 2 & 3 & 12 \\ 2 & -3 & 0 & 2 & -14 \end{array} \qquad A \cdot C = \begin{pmatrix} 3 & 12 \\ 2 & -14 \end{pmatrix}$$

$$\begin{array}{cc|cc} & & 2 & -1 \\ & & -1 & 3 \\ \hline 2 & -1 & 5 & -5 \\ -1 & 3 & -5 & 10 \end{array} \qquad B \cdot B = \begin{pmatrix} 5 & -5 \\ -5 & 10 \end{pmatrix}$$

$$
\begin{array}{rr|rrr}
 & & 1 & 1 & 2 \\
 & & 2 & -3 & 0 \\
\hline
2 & -1 & 0 & 5 & 4 \\
-1 & 3 & 5 & -10 & -2
\end{array}
\qquad
B \cdot A = \begin{pmatrix} 0 & 5 & 4 \\ 5 & -10 & -2 \end{pmatrix}
$$

$$
\begin{array}{rr|rrr}
 & & 1 & 1 & 2 \\
 & & 2 & -3 & 0 \\
\hline
1 & 2 & 5 & -5 & 2 \\
0 & 6 & 12 & -18 & 0 \\
1 & 2 & 5 & -5 & 2
\end{array}
\qquad
C \cdot A = \begin{pmatrix} 5 & -5 & 2 \\ 12 & -18 & 0 \\ 5 & -5 & 2 \end{pmatrix}
$$

$$
\begin{array}{rr|rr}
 & & 2 & -1 \\
 & & -1 & 3 \\
\hline
1 & 2 & 0 & 5 \\
0 & 6 & -6 & 18 \\
1 & 2 & 0 & 5
\end{array}
\qquad
C \cdot B = \begin{pmatrix} 0 & 5 \\ -6 & 18 \\ 0 & 5 \end{pmatrix}
$$

2.2

$$
\begin{array}{rrr|rr}
 & & & 1 & 2 \\
 & & & 0 & 6 \\
 & & & 1 & 2 \\
\hline
1 & 1 & 2 & 3 & 12 \\
2 & -3 & 0 & 2 & -14
\end{array}
\qquad
A \cdot C = \begin{pmatrix} 3 & 12 \\ 2 & -14 \end{pmatrix}
$$

$$
B^T = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}
$$

$$
\begin{aligned}
A \cdot C + 2B^T &= \begin{pmatrix} 3 & 12 \\ 2 & -14 \end{pmatrix} + \begin{pmatrix} 4 & -2 \\ -2 & 6 \end{pmatrix} \\
&= \begin{pmatrix} 7 & 10 \\ 0 & -8 \end{pmatrix}
\end{aligned}
$$

3 LGS

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ -3 & -4 & -8 \end{pmatrix}, \qquad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}, \qquad A{\cdot}B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{rcrcrcr|r|r} b_{1n} & + & 2b_{2n} & + & 3b_{3n} & = & 1 & 0 & 0 \\ 2b_{1n} & + & 3b_{2n} & + & 5b_{3n} & = & 0 & 1 & 0 \\ -3b_{1n} & - & 4b_{2n} & - & 8b_{3n} & = & 0 & 0 & 1 \end{array}$$

$$\begin{array}{rrr|rrr} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 5 & 0 & 1 & 0 \\ -3 & -4 & -8 & 0 & 0 & 1 \\ \hline 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 & 1 \\ \hline 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 2 & 1 & 3 & 0 & 1 \\ \hline 0 & 0 & -1 & -1 & 2 & 1 \end{array}$$

$$\begin{array}{rcl|rcl|rcl} -b_{31} & = & -1 & -b_{32} & = & 2 & -b_{33} & = & 1 \\ b_{31} & = & 1 & b_{32} & = & -2 & b_{33} & = & -1 \\ & & & & & & & & \\ b_{21} + b_{31} & = & 2 & b_{22} + b_{32} & = & -1 & b_{23} + b_{33} & = & 0 \\ b_{21} + 1 & = & 2 & b_{22} - 2 & = & -1 & b_{23} - 1 & = & 0 \\ b_{21} & = & 1 & b_{22} & = & 1 & b_{23} & = & 1 \\ & & & & & & & & \\ b_{11} + 2b_{21} + 3b_{31} & = & 1 & b_{12} + 2b_{22} + 3b_{32} & = & 0 & b_{13} + 2b_{23} + 3b_{33} & = & 0 \\ b_{11} + 2 \cdot 1 + 3 \cdot 1 & = & 1 & b_{12} + 2 \cdot 1 - 3 \cdot 2 & = & 0 & b_{13} + 2 \cdot 1 - 3 \cdot 1 & = & 0 \\ b_{11} + 5 & = & 1 & b_{12} - 4 & = & 0 & b_{13} - 1 & = & 0 \\ b_{11} & = & -4 & b_{12} & = & 4 & b_{13} & = & 1 \end{array}$$

$$B = \begin{pmatrix} -4 & 4 & 1 \\ 1 & 1 & 1 \\ 1 & -2 & -1 \end{pmatrix}$$