

Lösungen der Hausaufgabe Nr. 3 Lineare Algebra
Studiengang Network Computing
WS 2004/2005

Martin Grandrath (Matr. Nr.: 46375)

31. Oktober 2004

1 Mengenalgebraische Umformung

1.1

$$\begin{aligned}A \setminus (B \cup C) &= A \cap \overline{(B \cup C)} \\ &= A \cap (\overline{B} \cap \overline{C}) \\ &= (A \cap \overline{B}) \cap (A \cap \overline{C}) \\ &= (A \setminus B) \cap (A \setminus C)\end{aligned}$$

1.2

$$\begin{aligned}(A \setminus B) \setminus C &= (A \cap \overline{B}) \cap \overline{C} \\ &= A \cap \overline{C} \cap \overline{B} \cap \overline{C} \\ &= (A \setminus C) \cap (\overline{B} \setminus C)\end{aligned}$$

1.3

$$\begin{aligned}C \setminus (A \Delta B) &= C \setminus ((A \cap \bar{B}) \cup (\bar{A} \cap B)) \\&= C \cap \overline{((A \cap \bar{B}) \cup (\bar{A} \cap B))} \\&= C \cap (\overline{A \cap \bar{B}} \cap \overline{\bar{A} \cap B}) \\&= C \cap (\bar{A} \cup B) \cap (A \cup \bar{B}) \\&= ((C \cap \bar{A}) \cup (C \cap B)) \cap (A \cup \bar{B}) \\&= ((C \cap \bar{A}) \cap (A \cup \bar{B})) \cup ((C \cap B) \cap (A \cup \bar{B})) \\&= ((C \cap \bar{A} \cap A) \cup (C \cap \bar{A} \cap \bar{B})) \cup ((C \cap B \cap A) \cup (C \cap B \cap \bar{B})) \\&= (C \cap \bar{A} \cap \bar{B}) \cup (C \cap B \cap A) \\&= (C \cap \overline{A \cup B}) \cup (A \cap B \cap C) \\&= (C \setminus (A \cup B)) \cup (A \cap B \cap C)\end{aligned}$$

2 Mengensystem

$$\begin{aligned}\mathfrak{M} &= \left\{ \left[0, \frac{1}{n}\right] : n \in \mathbb{N} \setminus \{0\} \right\} \\ \bigcap \mathfrak{M} &= \{0\} \\ \bigcup \mathfrak{M} &= [0, 1]\end{aligned}$$

3 Kartesisches Produkt

3.1

$$\begin{aligned}(x, y) &\in (A \times (B \setminus C)) \\ \leadsto x &\in A \wedge y \in (B \setminus C) \\ \leadsto x &\in A \wedge y \in B \wedge y \notin C \\ \leadsto (x &\in A \wedge y \in B) \wedge (x \in A \wedge y \notin C) \\ \leadsto (x, y) &\in (A \times B) \wedge (x, y) \in (A \times \bar{C}) \\ \leadsto (x, y) &\in (A \times B) \setminus (A \times C)\end{aligned}$$

3.2

$$\begin{aligned} & (x, y) \in (A \cup B) \times (C \cap D) \\ \Leftrightarrow & x \in (A \cup B) \wedge y \in (C \cap D) \\ \Leftrightarrow & (x \in A \vee x \in B) \wedge (y \in C \wedge y \in D) \\ \Leftrightarrow & ((x \in A \wedge y \in C) \vee (x \in B \wedge y \in C)) \wedge y \in D \\ \Leftrightarrow & (x \in A \wedge y \in C \wedge y \in D) \vee (x \in B \wedge y \in C \wedge y \in D) \\ \Leftrightarrow & ((x \in A \wedge y \in C) \wedge (x \in A \wedge y \in D)) \vee ((x \in B \wedge y \in C) \wedge (x \in B \wedge y \in D)) \\ \Leftrightarrow & (x, y) \in ((A \times C) \cap (A \times D)) \vee (x, y) \in ((B \times C) \cap (B \times D)) \\ \Leftrightarrow & (x, y) \in ((A \times C) \cap (A \times D)) \cup ((B \times C) \cap (B \times D)) \end{aligned}$$