Lösungen der Hausaufgabe Nr. 2 Lineare Algebra
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1 Vereinfachung

$$
\begin{aligned}
& (x \to \overline{y}) \wedge \overline{\overline{x} \wedge \overline{(y \to z)}} \\
\equiv\ & (\overline{x} \vee \overline{y}) \wedge (x \vee (y \to z)) \\
\equiv\ & (\overline{x} \vee \overline{y}) \wedge (x \vee \overline{y} \vee z) \\
\equiv\ & (\overline{x} \vee \overline{y}) \wedge x \ \vee\ (\overline{x} \vee \overline{y}) \wedge \overline{y} \ \vee\ (\overline{x} \vee \overline{y}) \wedge z \\
\equiv\ & \overline{x} \wedge x \ \vee\ \overline{y} \wedge x \ \vee\ \overline{x} \wedge \overline{y} \ \vee\ \overline{y} \wedge \overline{y} \ \vee\ z \wedge \overline{x} \ \vee\ z \wedge \overline{y} \\
\equiv\ & \overline{y} \wedge x \ \vee\ \overline{y} \wedge \overline{x} \ \vee\ \overline{y} \ \vee\ \overline{x} \wedge z \ \vee\ \overline{y} \wedge z \\
\equiv\ & \overline{y} \wedge (x \vee \overline{x} \vee z) \ \vee\ \overline{y} \ \vee\ \overline{x} \wedge z \\
\equiv\ & \overline{y} \wedge z \ \vee\ \overline{y} \ \vee\ \overline{x} \wedge z \\
\equiv\ & \overline{y} \ \vee\ \overline{x} \wedge z
\end{aligned}
$$

2 Formalisieren

Nicht alle natürlichen Zahlen sind das Quadrat einer natürlichen Zahl

$$\sim \forall n \in \mathbb{N}\ \exists p \in \mathbb{N}\ (n = p^2)$$

3 Aussageformen

$$\exists x \left(H_1(x)\right) \wedge \exists x \left(H_2(x)\right) \tag{1}$$

$$\exists x \left(H_1(x) \wedge H_2(x)\right) \tag{2}$$

Es gilt (2) $\rightarrow$ (1) aber (1) $\nrightarrow$ (2)

Gegenbeispiel:

$$H_1(x) := (x = 5), \qquad H_2(x) := (x = 7)$$

4 Umformung

4.1

$$\begin{aligned} A \setminus (B \cup C) &= \{x \mid x \in A \wedge x \notin (B \cup C)\} \\ &= \{x \mid x \in A \wedge (x \notin B \wedge x \notin C)\} \\ &= \{x \mid (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)\} \\ &= \{x \mid x \in (A \setminus B) \wedge x \in (A \setminus C)\} \\ &= (A \setminus B) \cap (A \setminus C) \end{aligned}$$

4.2

$$\begin{aligned} (A \setminus B) \setminus C &= \{x \mid (x \in A \wedge x \notin B) \wedge x \notin C\} \\ &= \{x \mid (x \in A \wedge x \notin C) \wedge (x \notin B \wedge x \notin C)\} \\ &= \{x \mid (x \in (A \setminus C)) \wedge (x \in (\overline{B} \setminus C))\} \\ &= (A \setminus C) \cap (\overline{B} \setminus C) \end{aligned}$$

4.3

$$
\begin{aligned}
C \setminus (A \triangle B) &= \{x \mid x \in C \land x \notin (A \triangle B)\} \\
&= \{x \mid x \in C \land (x \in (A \cap B) \lor x \notin (A \cup B))\} \\
&= \{x \mid x \in C \land (x \in A \land x \in B \lor (x \notin A \land x \notin B))\} \\
&= \{x \mid x \in C \land x \in A \land x \in B \ \lor \ x \in C \land x \notin A \land x \notin B\} \\
&= \{x \mid x \in C \land x \in A \land x \in B \ \lor \ x \in C \land x \notin (A \cup B)\} \\
&= \{x \mid x \in (C \cap A \cap B) \ \lor \ x \in C \setminus (A \cup B)\} \\
&= (C \cap A \cap B) \cup (C \setminus (A \cup B)) \\
&= (C \setminus (A \cup B)) \cup (A \cap B \cap C)
\end{aligned}
$$