Lösungen der Übungsaufgaben Analysis 1 (Serie 2)
Studiengang Network Computing
WS 2004/2005

Martin Grandrath (Matr. Nr.: 46375)

23. Oktober 2004

1 Beträge und Ungleichungen

1.1

$$|2x-3| = 3$$

$$|2x-3| = \begin{cases} 3 & \text{für } 2x-3 \geq 0 \Rightarrow x \geq \frac{3}{2} \\ -3 & \text{für } 2x-3 < 0 \Rightarrow x < \frac{3}{2} \end{cases}$$

1. $\left(\mathbb{D}_1 = \left\{x : x \geq \frac{3}{2}\right\}\right)$

$$2x-3 = 3$$
$$2x = 6$$
$$x = 3 \qquad \Rightarrow \mathbb{L}_1 = \{3\}$$

2. $\left(\mathbb{D}_2 = \left\{x : x < \frac{3}{2}\right\}\right)$

$$2x-3 = -3$$
$$2x = 0$$
$$x = 0 \qquad \Rightarrow \mathbb{L}_2 = \{0\}$$

Ergebnis

$$\mathbb{L}_{ges} = \{0, 3\}$$

1.2

$$
\begin{aligned}
|x+1|-|2x-1| &= 1\\
|x+1| &= \begin{cases} x+1 & \text{für } x+1\geq 0 \Rightarrow x\geq -1\\ -x-1 & \text{für } x+1<0 \Rightarrow x<-1\end{cases}\\
|2x-1| &= \begin{cases} 2x-1 & \text{für } 2x-1\geq 0 \Rightarrow x\geq \frac{1}{2}\\ -2x+1 & \text{für } 2x-1<0 \Rightarrow x<\frac{1}{2}\end{cases}\\
\textbf{1.1} \quad & \left(\mathbb{D}_1=\left\{x: x\geq -1 \land x\geq \frac{1}{2}\right\}=\left\{x: x\geq \frac{1}{2}\right\}\right)\\
(x+1)-(2x-1) &= 1\\
x+1-2x+1 &= 1\\
-x+2 &= 1\\
x &= 1 \qquad \Rightarrow \mathbb{L}_1=\{1\}\\
\textbf{1.2} \quad & \left(\mathbb{D}_2=\left\{x: x\geq -1 \land x< \frac{1}{2}\right\}=\left\{x: -1\leq x< \frac{1}{2}\right\}\right)\\
(x+1)-(-2x+1) &= 1\\
x+1+2x-1 &= 1\\
3x &= 1\\
x &= \frac{1}{3} \qquad \Rightarrow \mathbb{L}_2=\left\{\frac{1}{3}\right\}\\
\textbf{2.1} \quad & \left(\mathbb{D}_3=\left\{x: x< -1 \land x\geq \frac{1}{2}\right\}=\emptyset\right)\\
& \qquad \Rightarrow \mathbb{L}_3=\emptyset\\
\textbf{2.2} \quad & \left(\mathbb{D}_4=\left\{x: x< -1 \land x< \frac{1}{2}\right\}=\{x: x<-1\}\right)\\
(-x-1)-(-2x+1) &= 1\\
-x-1+2x-1 &= 1\\
x &= 3 \qquad \Rightarrow \mathbb{L}_4=\emptyset\\
\textbf{Ergebnis} & \\
\mathbb{L}_{ges} &= \mathbb{L}_1\cup\mathbb{L}_2\cup\mathbb{L}_3\cup\mathbb{L}_4=\left\{\frac{1}{3},1\right\}
\end{aligned}
$$

1.3

$$x - 1 < \frac{2x-2}{x-4} \qquad \mathbb{D} = \{x : x \neq 4\}$$

$$\begin{cases} (x-1)(x-4) < 2x-2 & \text{für } x-4>0 \Rightarrow x>4 \\ (x-1)(x-4) > 2x-2 & \text{für } x-4<0 \Rightarrow x<4 \end{cases}$$

1. $(\mathbb{D}_1 = \{x : x > 4\})$

$$\begin{aligned} (x-1)(x-4) &< 2x-2 \\ x^2 - 5x + 4 &< 2x - 2 \\ x^2 - 7x + 6 &< 0 \end{aligned}$$

2. $(\mathbb{D}_2 = \{x : x < 4\})$

$$\begin{aligned} (x-1)(x-4) &> 2x-2 \\ x^2 - 5x + 4 &> 2x - 2 \\ x^2 - 7x + 6 &> 0 \end{aligned}$$

Nullstellen bestimmen

$$\begin{aligned} x_{1/2} &= -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} \\ &= \frac{7}{2} \pm \sqrt{\frac{49}{4} - 6} \\ &= \frac{7}{2} \pm \sqrt{\frac{49-24}{4}} \\ &= \frac{7}{2} \pm \frac{5}{2} \\ x_1 &= 6 \\ x_2 &= 1 \end{aligned}$$

Ergebnis

$$\begin{aligned} \mathbb{L}_1 &= \{x : x > 4 \wedge 1 < x < 6\} = \{x : 4 < x < 6\} \\ \mathbb{L}_2 &= \{x : x < 4 \wedge (x < 1 \vee x > 6)\} = \{x : x < 1\} \\ \mathbb{L}_{ges} &= \mathbb{L}_1 \cup \mathbb{L}_2 = \{x : x < 1 \vee 4 < x < 6\} \end{aligned}$$

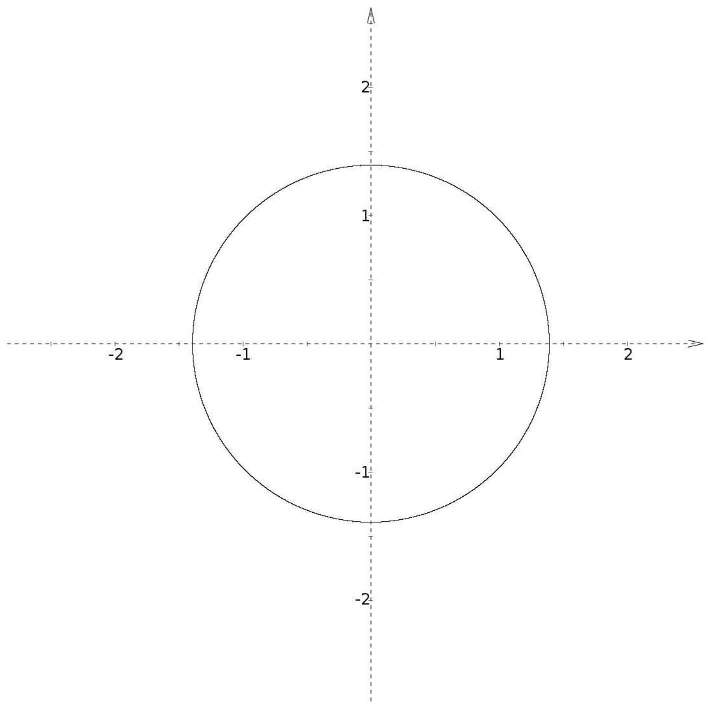
2 Punkte im Koordinatensystem

2.1

$$
\begin{aligned}
x^2 + y^2 &= 2 \\
y^2 &= -x^2 + 2 \\
|y| &= \sqrt{-x^2 + 2} \\
y &= \begin{cases} \sqrt{-x^2+2} & \text{für } y \geq 0 \\ -\sqrt{-x^2+2} & \text{für } y < 0 \end{cases}
\end{aligned}
$$

Definitionsbereich

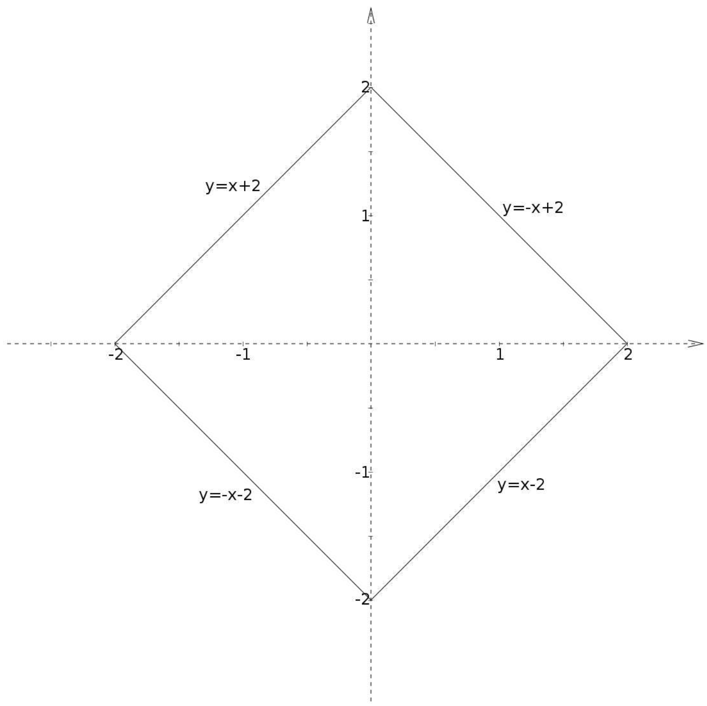
$$
\begin{aligned}
-x^2 + 2 &\geq 0 \\
x^2 &\leq 2 \\
|x| &\leq \sqrt{2} \\
\mathbb{D} &= \left\{ x : \ -\sqrt{2} \leq x \leq \sqrt{2} \right\}
\end{aligned}
$$



2.2

$$|x| + |y| = 2$$

$$\begin{cases} y = -x + 2 & \text{für } x \geq 0 \wedge y \geq 0 \\ y = x - 2 & \text{für } x \geq 0 \wedge y < 0 \\ y = x + 2 & \text{für } x < 0 \wedge y \geq 0 \\ y = -x - 2 & \text{für } x < 0 \wedge y < 0 \end{cases}$$

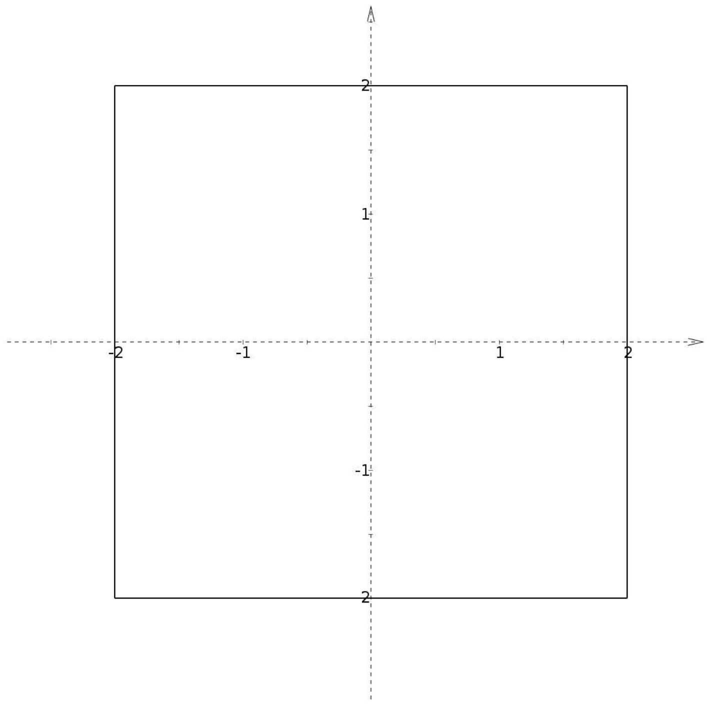


2.3

$$\max(|x|, |y|) = 2$$

$$\begin{cases} |x| = 2 & \text{für } |y| \leq 2 \\ |y| = 2 & \text{für } |x| \leq 2 \end{cases}$$

$$\begin{cases} x = 2 \vee x = -2 & \text{für } -2 \leq y \leq 2 \\ y = 2 \vee y = -2 & \text{für } -2 \leq x \leq 2 \end{cases}$$



3 Supremum und Infimum

$$A := \left\{ \frac{3}{n} : n \in \mathbb{N} \right\} \subset \mathbb{R}$$
$$\sup A = \max A = 3$$
$$\inf A = 0$$

Ein Minimum von A existiert nicht.

4 Summen

4.1

$$1 + 5 + 9 + 13 + 17 + \cdots + 2005 = \sum_{n=0}^{501} (4n + 1)$$

4.2

$$\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \cdots + \frac{9999}{10001} = \sum_{n=1}^{9999} \frac{n}{n+2}$$

5 Indextransformation

$$\sum_{i=1}^{n} a_i = \sum_{r=6}^{\boldsymbol{n+5}} a_{\boldsymbol{r-5}} = \sum_{\boldsymbol{\nu=0}}^{\boldsymbol{n-1}} a_{\nu+1} = \sum_{\boldsymbol{q=3}}^{n+2} a_{q\boldsymbol{-2}} = \sum_{\rho=\boldsymbol{2}}^{\boldsymbol{n+1}} a_{\rho-1} = \sum_{\alpha=\boldsymbol{n}}^{\boldsymbol{1}} a_{n-\alpha+1}$$

6 Zusatzaufgabe

Definition der Fibonacci-Zahlen

$$F_{n+1} = F_n + F_{n-1}$$

Zu zeigen:

$$F_n{}^2 - (F_{n+1} \cdot F_{n-1}) = (-1)^{n-1} \qquad \text{für } n \in \mathbb{N} : n \geq 2$$

Induktionsanfang ($n_1 = 2$)

$$1^2 - 1 \cdot 2 = (-1)^1$$

Induktionsschluss

$$\begin{aligned}
(F_n + F_{n-1})^2 - (F_{n-1} + F_{n-2}) \cdot (F_{n+1} + F_n) &= (-1)^n \\
F_n{}^2 + 2F_nF_{n-1} + F_{n-1}{}^2 - (F_{n-1} + F_n - F_{n-1}) \cdot (F_{n+1} + F_{n+1} - F_{n-1}) &= (-1)^n \\
F_n{}^2 + 2F_nF_{n-1} + F_{n-1}{}^2 - (F_{n+1} - F_{n-1}) \cdot (2F_{n+1} - F_{n-1}) &= (-1)^n \\
F_n{}^2 + 2F_nF_{n-1} + F_{n-1}{}^2 - 2F_{n+1}{}^2 + F_{n-1}F_{n+1} + 2F_{n+1}F_{n-1} - F_{n-1}{}^2 &= (-1)^n \\
F_n{}^2 + 2F_nF_{n-1} - 2F_{n+1}{}^2 + 3F_{n-1}F_{n+1} &= (-1)^n \\
F_n{}^2 + 2F_{n-1}(F_{n+1} - F_{n-1}) - 2F_{n+1}{}^2 + 3F_{n-1}F_{n+1} &= (-1)^n \\
F_n{}^2 + 2F_{n-1}F_{n+1} - 2F_{n-1}{}^2 - 2F_{n+1}{}^2 + 3F_{n-1}F_{n+1} &= (-1)^n \\
F_n{}^2 + 5F_{n-1}F_{n+1} - 2F_{n-1}{}^2 - 2F_{n+1}{}^2 &= (-1)^n \\
F_n{}^2 + 5F_{n-1}F_{n+1} - 2(F_{n+1} - F_n)^2 - 2F_{n+1}{}^2 &= (-1)^n \\
F_n{}^2 + 5F_{n-1}F_{n+1} - 2(F_{n+1}{}^2 - 2F_{n+1}F_n + F_n{}^2) - 2F_{n+1}{}^2 &= (-1)^n \\
F_n{}^2 + 5F_{n-1}F_{n+1} - 2F_{n+1}{}^2 + 4F_{n+1}F_n - 2F_n{}^2 - 2F_{n+1}{}^2 &= (-1)^n \\
-F_n{}^2 + 5F_{n-1}F_{n+1} - 4F_{n+1}{}^2 + 4F_{n+1}F_n &= (-1)^n \\
-F_n{}^2 + 5F_{n-1}F_{n+1} - 4F_{n+1}(F_{n+1} - F_n) &= (-1)^n \\
-F_n{}^2 + 5F_{n-1}F_{n+1} - 4F_{n+1}F_{n-1} &= (-1)^n \\
-F_n{}^2 + F_{n-1}F_{n+1} &= (-1)^n \\
F_n{}^2 - F_{n-1}F_{n+1} &= (-1)^{n-1}
\end{aligned}$$